3.20.1. Semantic Problems: Validity, Consistency, and Counterexample Sets

1. It was noted in "*3.19. Features of Validity"* that any sentence follows validly from itself. Use **counterexample sets** and **(in)consistency** to explain why this is so.

2. It was also noted that, beginning with a valid argument, adding further premises always yields a (bigger) valid argument. For example, since this argument is (famously) valid,

1. $(P \vee Q)$
2. $\sim P$

$\therefore Q$

the following argument is also valid.

1. $(P \vee Q)$
2. $\sim P$
3. X

$\therefore Q$

Use **counterexample sets** and **(in)consistency** to explain why adding further premises always leaves the (larger) argument valid.

(*Hint: use the features of consistent and inconsistent sets discussed in "3.17. Semantic Concepts"*)

3. It was also noted that an argument whose conclusion is a tautology is sure to be valid. Use **counterexample sets** and **(in)consistency** to explain why this is so.

4. It was also noted that an argument with inconsistent premises is sure to be valid. Use **counterexample sets** and **(in)consistency** to explain why this is so.

5. Show that if an argument is invalid, then removing one or more premises will always yield a (smaller) argument which is also invalid.

(*Hint: use the features of consistent and inconsistent sets discussed in “3.17. Semantic Concepts”*)

6. (a) *3.18.1* Problem C showed that a consistent sentence never entails a contradiction. (Equivalently: that **the only (sort of) sentence that entails a contradiction is a contradiction**.)

That means the following principle holds, for one-premise arguments.

(1) **A contradiction follows validly only from a contradiction**.

Explain how Principle (1) can be strengthened to hold for arguments with any number of premises (so long as those sentences form an inconsistent set) – yielding Principle (2).

(2) A contradiction follows validly only from an inconsistent set of sentences. (Equivalently: **no contradiction follows from a consistent set of sentences**.)

(b) Use the features of inconsistent premises and validity discussed in *3.20* to strengthen (2) into Principle (3).

(3) A contradiction follows validly from every inconsistent set of premises, and only from an inconsistent set of premises.